Math 1A Worksheet 22

April 2nd, 2008

1. By taking derivatives, show that $\sin^{-1}(\tanh x) = \tan^{-1}(\sinh x)$. [Note: remember, to show $f = g$, it is NOT enough to show $f' = g'$!]
2. Show that $x^{1001} + 10x + 1$ has exactly one real root.
3. Use Rolle's theorem to show that a polynomial of degree 3 has at most 3 real roots.
4. Along the same lines: suppose that f is twice differentiable on all of $\mathbb{R}$ and has three real roots. Show that f'' has at least one real root.
5. Show that if $0 \leq a < b \leq \frac{\pi}{4}$, then $\tan b - \tan a < 2(b - a)$.