

Math 1A Worksheet 20

March 19th, 2008

1. Find the following limits:

a)

$$\lim_{x \rightarrow \infty} \sin^{-1}(\tanh(x))$$

b)

$$\lim_{x \rightarrow \infty} \cosh(x) - \sinh(x)$$

2. Find, with proof, the following derivatives:

a) $\frac{d}{dx} \cosh x$,

b) $\frac{d}{dx} \cosh^{-1}(x)$.

3. Prove that $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$ using the following steps:

a) Show that $\cosh(\sinh^{-1}(x)) = \sqrt{1 + x^2}$.

b) Prove the formula $\cosh x + \sinh x = e^x$.

c) Substitute $\sinh^{-1}(x)$ for x in equation b) and simplify.

4. Let

$$f(x) = \begin{cases} x \arctan(1/x) & , \quad x \neq 0 \\ 0 & , \quad x = 0. \end{cases}$$

Is f continuous at 0? Is f differentiable at 0?

5. Suppose a_1, a_2, \dots, a_{14} are some 14 integers. Show that

$$\prod_{1 \leq i < j \leq 14} (a_i - a_j)$$

is divisible by $2002 = 2 \cdot 7 \cdot 11 \cdot 13$.