

## Math 1A Worksheet 2

January 24th, 2008

1. Is there a function all of whose values are equal to each other? If so, graph your answer, if not, explain why.
2. Consider the polynomial functions

$$f(x) = -x^2 + 1, \quad g(x) = (x - 1)^2, \quad h(x) = x^3.$$

- i) Find all  $x$  such that  $f(x) \leq 2$ . Do the same for  $g(x)$  and  $h(x)$ . Write your answers in interval notation and draw them on the graphs of the functions.
  - ii) Find all  $x$  such that  $|f(x)| < 2$ . Do the same for  $|g(x)|$  and  $|h(x)|$ .
  - iii) Can you find an *upper bound* for  $f$ ? That is, can you find a number  $M$  such that  $f(x) \leq M$  for all  $x$ ? What about  $g$  and  $h$ ?
  - iv) Can you find a *lower bound* for  $f$ ? I leave it to you to define this. What about  $g$  and  $h$ ?
  - v) What about finding upper bounds for  $f$  *restricted* to  $[-1, 1]$ ? That is, can you find  $M$  such that  $f(x) \leq M$  for all  $x$  in  $[-1, 1]$ ? How about  $g$  and  $h$ ? Can you find lower bounds for  $f$ ,  $g$ , and  $h$  on  $[-1, 1]$ ?
3. Let  $f$  and  $g$  be two functions. Prove or give a counterexample: If  $M$  is an upper bound for  $|f(x)|$  and  $N$  is an upper bound for  $|g(x)|$ , then

$$|f(x) + g(x)| \leq M + N$$

for all  $x$ . *Hint: triangle inequality!*

4. Graph the functions below. Find their maximum and minimum values, if they exist.
- i)  $f(x) = \frac{1}{x}$
  - ii)  $f(x) = -x^2 + 1$
  - iii)  $f(x) = \sin x - 1$
  - iv)  $f(x) = \sin(x - 1)$
  - v)  $f(x) = \sin\left(\frac{1}{x}\right)$ .