Math 1A Worksheet 14

March 5th, 2008

1. a) Using the $\lim_{h\to 0}$ definition of the derivative, show that the derivative of an exponential function is proportional to the function itself. That is, if $f(x) = a^x$, show that there is a constant k such that $f'(x) = k \cdot a^x = k \cdot f(x)$. What is the value of the constant k for each function a^x?

   b) How is the exponential function $f(x) = e^x$ different from other exponential functions? (Hint: how is the derivative of e^x different from that of 2^x or 3^x?)

2. Is the function $f(x) = x + \sin x$ one-to-one? [Hint: look at f'(x). We used a particular trick in our last class to show that $x = \sin x$ has only one solution. Repeat this trick to show that $f(x)$ is strictly increasing. Explain how this solves our problem.]

3. a) Suppose $f(x)$ is an invertible function and (a, b) lies on the graph of f. What point must lie on the graph of $y = f^{-1}(x)$?
   b) Graph the function $f(x) = x^3$ and its inverse on the same axes. Which line do you reflect the graph of f across to get the graph of f^{-1}?
   c) What do you know about the points where the graphs of f and f^{-1} intersect?

4. Evaluate $\lim_{x\to\pi} \frac{e^{\sin x}-1}{x-\pi}$.