

Math 1A Worksheet 13

March 3rd, 2007

1. Without graphing the entire curve $\sin x$, graph the points on the curve $y = \sin x$ corresponding to $x = 0, \pi/2, \pi, 3\pi/2$, and 2π . Now find the slopes of the tangent lines to $y = \sin x$ at these four points, and sketch these tangent lines. What do you notice?
2. Write down the equation for the area A of a circle in terms of its radius r . Then:
 - a) Use this to compute dA in terms of dr .
 - b) Draw two concentric circles, one with radius r and the second with radius $r + \Delta r$ where Δr is reasonably small.
 - c) Shade the area on your drawing which represents ΔA . Calculate ΔA in terms of r and Δr .
 - d) Explain why dA is a good approximation to ΔA .
3. (Quadratic Approximation) Let f be a function which is twice-differentiable at a point a . Just as we can approximate $f(x)$ near a by a line (i.e. a degree-1 polynomial), we can also approximate it by a quadratic function (i.e. a degree-2 polynomial). We find this approximation as follows: write $P(x) = A + B(x - a) + C(x - a)^2$. We want to have $P(a) = f(a)$, $P'(a) = f'(a)$, and $P''(a) = f''(a)$. Find a formula for A , B , and C in terms of $f(a)$, $f'(a)$, and $f''(a)$.
4. Apply the previous problem to find a quadratic approximation for $f(x) = \sin x$ near $a = \pi/2$. Graph $f(x)$, the linear approximation to $f(x)$ at a , and the quadratic approximation to $f(x)$ at a all on the same graph.