

Math 1A Worksheet 12

February 27th, 2007

Easier Problems - Do These First

1. Consider the curve defined by $y^3 + xy + x^4 = 11$. Find the slope of the tangent to this curve at $(1, 2)$. Find a formula for $\frac{d^2y}{dx^2}$ in terms of only x and y .
2. True or false?

If P is a polynomial of degree 6, then $P^{(7)} = 0$.

(Remember that $P^{(7)}$ is the seventh derivative of P .) If the statement is true, explain why. If false, give a counterexample and correct the statement. Once you have a true statement, come up with a more general statement that is also true.

Harder Problems - Computational

3. If n is a positive integer, prove that $\frac{d}{dx}(\sin^n x \cos nx) = n \sin^{n-1} x \cdot \cos(n+1)x$. Find a similar formula for $\frac{d}{dx}(\cos^n x \cos nx)$.
4. Find the following derivatives by finding a pattern:
 - a) $\frac{d^{99}}{dx^{99}} \sin x$,
 - b) $\frac{d^{100}}{dx^{100}} \cos x$,
 - c) $\frac{d^{50}}{dx^{50}} \cos 2x$,
 - d) $\frac{d^{35}}{dx^{35}} x \sin x$.

Harder Problem - Conceptual

5. How many solutions are there to $x = \sin x$? [Hint: to prove your solution, try a derivative argument!]