Math 1A Worksheet 10

September 24th, 2007

1. In preparation for the chain rule, practice with composition. If $k(x) = f(g(h(x)))$, complete the following table:

If $f(x) = x^2$,	$g(x) = \sin(x)$,	$h(x) = \cos(4x)$,	then $k(x) =$ ____.
If $f(x) = \frac{1}{x}$,	$g(x) =$ ____,	$h(x) = (12 - x^2)$,	then $k(x) = (12 - x^2)^{-2}$.
If $f(x) =$ ____,	$g(x) = x$,	$h(x) =$ ____,	then $k(x) = \cos(\tan x)$.
If $f(x) =$ ____,	$g(x) =$ ____,	$h(x) =$ ____,	then $k(x) = [1 - (3x - 2)^3]^4$.

2. Using the quotient rule and the derivatives of $\sin x$ and $\cos x$, show that $\frac{d}{dx} \tan x = \sec^2 x$.

3. Find a degree-2 polynomial $p(x)$ whose graph passes through the point $(3, 6)$ and which has a horizontal tangent at the point $(2, 8)$.

4. Let $f(x) = \sqrt[3]{x}$. Use the $x - a$ formula for the derivative to find $f'(a)$ when $a \neq 0$. The same calculation should show that $f'(0)$ does not exist. Draw a graph of f and attempt to explain why this is the case. [Note: This is hard, but it will be easier if you remember that $b^3 - c^3 = (b - c)(b^2 + bc + c^2)$.]

5. Let
$$f(x) = \left\{ \begin{array}{ll} x^2 \sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{array} \right\}.$$
Find $f'(x)$. (Note: you *cannot* use the product rule. Why not?) Where is $f'(x)$ continuous?

6. Let n be a positive integer. Find
$$\lim_{x \to 1} \frac{x^n - 1}{x - 1}.$$
(There's an easy way and a hard way...)

7. Find all prime numbers p such that $p + 2$ and $p + 4$ are also prime.