Math 1A Quiz 5 Solutions

March 6th, 2008

1. A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point $(2,3)$, the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

   Solution: Differentiate both sides with respect to t. We have $\frac{dy}{dt} = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2 \cdot \frac{dx}{dt}$. Plugging in $x = 2, y = 3, \frac{dy}{dt} = 4$ and solving for $\frac{dx}{dt}$ gives $\frac{dx}{dt} = 2$.

2. Let $f(x) = e^x \cdot (\sin x)^x$. Find $f'(\frac{\pi}{2})$.

   Solution 1: We have $(\sin x)^x = e^{x \ln(\sin x)}$, so $e^x (\sin x)^x = e^{x + x\ln(\sin x)}$. Differentiating using chain rule gives:

$$\begin{aligned} f'(x) &= e^{1+x\ln(\sin x)} \cdot \frac{d}{dx}(x + x\ln(\sin x)) \\ &= e^{1+x\ln(\sin x)} \cdot (1 + \ln(\sin x) + x\frac{\cos x}{\sin x}) \\ &= e^x(\sin x)^x \cdot (1 + \ln(\sin x) + x\frac{\cos x}{\sin x}) \end{aligned}$$

   Plugging in $x = \frac{\pi}{2}$, we get

$$\begin{aligned} f'(\frac{\pi}{2}) &= e^{\frac{\pi}{2}} 1^{\frac{\pi}{2}} \cdot (1 + \ln(1) + \frac{\pi}{2} \cdot \frac{0}{1}) \\ &= e^{\frac{\pi}{2}}. \end{aligned}$$

   Solution 2: Take logarithms of both sides of $y = e^x(\sin x)^x$, to get

$$\ln y = \ln(e^x(\sin x)^x) = \ln(e^x) + \ln((\sin x)^x) = x + x\ln(\sin x).$$

   Now take derivatives of both sides to get

$$\frac{1}{y}y' = 1 + \ln(\sin x) + x\frac{\cos x}{\sin x}.$$

   Multiplying both sides by $y = e^x(\sin x)^x$ gives exactly the same derivative formula as in solution 1, and the rest follows indentically.