Math 1A Quiz 4 Solution

February 29th, 2008

Name ____________________ SID ________________

1. Show that the sum of the x- and y-intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c.

Solution: We use implicit differentiation to find y': differentiating $\sqrt{x} + \sqrt{y} = \sqrt{c}$ gives $\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$. Solving for y', we get $y' = -\frac{\sqrt{y}}{\sqrt{x}}$.

Now, let (a, b) be a point on this curve, so $\sqrt{a} + \sqrt{b} = \sqrt{c}$. At this point, we get $y' = -\frac{\sqrt{b}}{\sqrt{a}}$, so the equation to the tangent line at this point (using point-slope form) is $(y-b) = -\frac{\sqrt{b}}{\sqrt{a}}(x-a)$. To find the x-intercept, we plug in $y = 0$ and solve for x, which gives $x = a + \frac{\sqrt{a}}{\sqrt{b}}b = a + \sqrt{a}\sqrt{b}$. Similarly, to solve for the y-intercept, we set $x = 0$ and solve for y, which gives $y = b + \sqrt{a}\sqrt{b}$. Thus thus sum of the x- and y-intercepts is

$$a + 2\sqrt{a}\sqrt{b} + b = (\sqrt{a} + \sqrt{b})^2.$$

Since $\sqrt{a} + \sqrt{b} = \sqrt{c}$, the sum of the x- and y-intercepts is $\sqrt{c}^2 = c$.

Note: The most-missed point on this problem was that you must plug in a and b into the formula for y'. The reason is that the formula $y' = -\frac{\sqrt{y}}{\sqrt{x}}$ tells you the slope of the tangent at a given point (x, y), but you have to actually pick a point and plug in the values to get the slope at that point. Otherwise the "tangent line equation" which you'll come up with won't even be the equation for a line.

2. Let $f(x) = (1 + \sin(\cos(\sin(2x))))^{100}$. Find $f'(x)$.

Solution: Many applications of chain rule give:

$$100 \cdot (1 + \sin(\cos(\sin(2x))))^{99} \cdot \cos(\cos(\sin(2x))) \cdot (-\sin(\sin(2x))) \cdot \cos(2x) \cdot 2.$$