Math 1A Quiz 10 Solutions

April 25th, 2008

1. Find the most general antiderivatives of each of the following functions:

   (a) $f(x) = \frac{x^4+3\sqrt{x}}{x^2}$

   (b) $g(x) = \frac{4}{\sqrt{1-x^2}}$

   (c) $h(x) = 2\cos x + \sec^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

   Solution to a): Distribute the denominator. We get

   $$f(x) = x^2 + 3x^{-3/2}.$$

   Now use the power rules for antidifferentiation to get

   $$F(x) = \frac{x^3}{3} - 6x^{-1/2} + C.$$

   Solution to b): The antiderivative is

   $$G(x) = 4\sin^{-1}(x) + C,$$

   by memorization.

   Solution to c): The antiderivative is

   $$H(x) = 2\sin x + \tan x + C.$$

2. (a) Write an expression for $\int_1^3 t^2 dt$ as a limit of Riemann sums. The Riemann sums you use should divide the interval $[1,3]$ into n pieces of equal width, and you should use the right-hand endpoints for your x_i^*.

   (b) Evaluate the Riemann sum you wrote down in part (a) when $n = 4$.

Solution to a): We divide the interval $[1,3]$ into n equal pieces, of length $\frac{3-1}{n} = \frac{2}{n}$. The ith piece of this partition has right-hand endpoint $1 + i\frac{2}{n}$, so according to the instructions in the problem we have $\Delta x = \frac{2}{n}$ and $x_i^* = 1 + i\frac{2}{n}$. The resulting Riemann sum is:

$$\sum_{i=1}^{n} f(x_i^*) \cdot \Delta x = \sum_{i=1}^{n} \left(1 + i\frac{2}{n}\right)^2 \cdot \frac{2}{n}.$$

And the integral is the limit of the Riemann sums:

$$\int_1^3 t^2 dt = \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + i\frac{2}{n}\right)^2 \cdot \frac{2}{n}.$$

Solution to b): Just plug in $n = 4$ into our Riemann sum. We get:

$$\begin{aligned}
\sum_{i=1}^{4} \left(1 + \frac{i}{2}\right)^2 \cdot \frac{1}{2} &= \frac{1}{2}\left[\left(1+\frac{1}{2}\right)^2 + \left(1+\frac{2}{2}\right)^2 + \left(1+\frac{3}{2}\right)^2 + \left(1+\frac{4}{2}\right)^2\right] \\
&= \frac{1}{2}\left(\frac{9}{4} + \frac{16}{4} + \frac{25}{4} + \frac{36}{4}\right) \\
&= \frac{43}{4}.
\end{aligned}$$