

Math 1A Practice Final
May 9th, 2008

Name _____ SID _____

1. The curves $y = x^2$, $y = (x - 2)^2$, and $y = 0$ bound a region D . Using the method of your choice, find the volume of the solid you obtain by rotating this region about the y -axis.

2. Find the area of the region that lies between the curves $y = \cos x$ and $y = \sin 2x$ and between $x = 0$ and $x = \frac{\pi}{2}$. [Hint: $\cos \frac{\pi}{6} = \sin \frac{\pi}{3}$.]

3. Let $f(x) = \sin x + 3x$, and let $g(x)$ be the inverse of $f(x)$.

(a) Find $g(3\pi)$.

(b) Find $\int_0^{3\pi} g(x) dx$.

4. Listed below are five properties of functions. For each, either give an example of a function with the description, or explain why no such function can exist.
- (a) A continuous function with domain \mathbb{R} which is not everywhere differentiable.

 - (b) A continuous function with domain \mathbb{R} which has no antiderivative.

 - (c) An invertible function with domain \mathbb{R} which is not everywhere continuous.

 - (d) A strictly increasing function which is not invertible.

 - (e) An everywhere differentiable function which is not invertible.

5. Simplify/evaluate the following expressions:

a) $\int_0^4 e^{\sqrt{x}} dx$. [Hint: use the substitution $u = \sqrt{x}$.]

b) $\frac{d^2}{dx^2} \int_{\cos x}^1 \left(\int_0^t \sqrt{1+u^4} du \right) dt$. [Note: here “simplify” means “get rid of the derivative.” Your answer will still have an integral in it.]

c) $\lim_{n \rightarrow \infty} \frac{4\pi}{n} \sum_{i=1}^n \sin\left(\frac{i\pi}{n}\right)$.

d) $\int_{-1}^{\sqrt{2}/2} \sqrt{4-x^2} dx$.