Math 1A Worksheet 9

September 19th, 2007

1. Using the limit laws and the identity $\lim_{x\to 0} \frac{\sin x}{x} = 1$, find
$$\lim_{x\to 0} \frac{\sin x}{x + \tan x}.$$

2. Give examples of the following:
a) Two functions f and g such that both f and g are continuous nowhere, but $f + g$ is continuous everywhere.

b) Two everywhere continuous functions f and g such that f and g are both not differentiable at 0, but $f + g$ is differentiable at 0.

c) Two functions f and g such that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ exist, but
$$\lim_{x\to 0} (f \circ g)(x)$$
does not.

d) Two functions f and g such that $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to\infty} g(x) = -\infty$, and
$$\lim_{x\to\infty} (f \circ g)(x) = 0.$$

3. Let $f(x) = \sqrt[3]{x}$. Use the $x - a$ formula for the derivative to find the $f'(a)$ when $a \neq 0$. The same calculation should show that $f'(0)$ does not exist. Draw a graph of f and attempt to explain why this is the case.
[Note: This is hard, but it will be easier if you remember that $b^3 - c^3 = (b - c)(b^2 + bc + c^2)$.]

4. Let f and g be two functions, both of which are differentiable at some point a. Using the limit laws and the limit definition of the derivative, show that $f+g$ is differentiable at a and that $(f+g)'(a) = f'(a)+g'(a)$.

5. Suppose we color the x, y-plane two colors: each point (x, y) is colored either red or blue. Show that there exist two points of the same color which are distance exactly 1 apart.

   Suppose now that we color the x, y-plane three colors: each point (x, y) is colored either red, blue, or green. Show that there still exist two points of the same color which are distance exactly 1 apart.