

Math 1A Worksheet 7

September 12th, 2007

1. Find

$$\lim_{x \rightarrow \infty} \frac{x^5 + 46x^4 + 9x^2 + 7}{7x^5 + 26x^3 + 8x}.$$

Using the limit laws, justify your answer.

2. Let

$$f(x) = \left\{ \begin{array}{ll} 0 & , \quad x \text{ irrational} \\ 1 & , \quad x \text{ rational} \end{array} \right\}.$$

and let

$$g(x) = x^2 f(x).$$

Find $g'(0)$.

[Note: for those who know the “product rule” for derivatives: why does this rule **not** apply here?]

3. Give examples of functions which are:
- Differentiable only at 0.
 - Continuous only at 0, but not differentiable at 0.
 - Continuous everywhere, and differentiable everywhere except -1 and 1 .
 - Differentiable only at -1 and 1 . [Hint for d): use a function like the one in Problem 2. You will probably have to do something piecewise.]
4. Let f be a continuous function from $[0, 1]$ to $[0, 1]$ (i.e. $[0, 1]$ is the domain of f , and $0 \leq f(x) \leq 1$ for each x in the domain of f). Show that f has a *fixed point*, that is, that there exists some a with $0 \leq a \leq 1$ such that $f(a) = a$. [Hint: apply the Intermediate Value Theorem to $f(x) - x$.]

Is the same thing true if f is any continuous function from the real numbers to the real numbers?

5. Explain the following in terms of the Intermediate Value Theorem:
At any instant, there is a point on the Earth's equator which has exactly the same temperature as the point on the opposite side of the Earth.