

# Math 1A Worksheet 5

September 10th, 2007

1. Calculate

$$\lim_{x \rightarrow -3^+} \frac{x^2 + 4x + 3}{|x + 3|}.$$

Explain your answer carefully.

2. Consider the function

$$f(x) = \left\{ \begin{array}{ll} 0 & , \quad x \text{ irrational} \\ x & , \quad x \text{ rational} \end{array} \right\}.$$

- a) Does

$$\lim_{x \rightarrow 0} f(x)$$

exist? If so, what is the limit? Is  $f$  continuous at 0?

- b) Let  $a$  be a nonzero real number. Does

$$\lim_{x \rightarrow a} f(x)$$

exist?

- c) For which real numbers  $a$  is  $f(x)$  continuous at  $a$ ?

3. The greatest integer function  $[[x]]$  is defined by: for each real number  $a$ ,  $[[a]]$  is the largest integer  $n$  such that  $n \leq a$ . So, for example,  $[[3/2]] = 1$ , since  $1 \leq 3/2$  but  $2 > 3/2$ .

- a) Let  $n$  be an integer. What is

$$\lim_{x \rightarrow n^+} [[x]]?$$

What about the limit from below? Does

$$\lim_{x \rightarrow n} \lfloor x \rfloor$$

exist?

b) Now, let  $a$  be a real number that is not an integer. Does

$$\lim_{x \rightarrow a} \lfloor x \rfloor$$

exist? What is it?

c) From a) and b), for which real numbers is  $\lfloor x \rfloor$  continuous?

d) Calculate

$$\lim_{x \rightarrow 1} \frac{x^2 - \frac{x}{2} - \frac{1}{2}}{x - 1}.$$

[Hint: the numerator factors!]

e) Find

$$\lim_{x \rightarrow 1} \left[ \left[ \frac{x^2 - \frac{x}{2} - \frac{1}{2}}{x - 1} \right] \right].$$