Math 1A Worksheet 4

September 5th, 2007

1. Suppose $f(x)$ is some increasing function with $f(3) = 2$ and $f(0) = -1$. If this is confusing, draw a graph of a function which has these properties. Find a $\delta > 0$ so that whenever $|x - 2| \leq \delta$, we have $-1 \leq f(x) \leq 2$. For the δ you have chosen, is it true that if $|x - 2| \leq \delta$, then $|f(x) - f(2)| \leq 3$?

2. Find a $\delta > 0$ such that whenever $|x - \frac{\pi}{2}| < \delta$, we have $|sinx - 1| < \frac{1}{2}$. If you are stuck, draw a graph of $sinx$, and remember how to turn an inequality involving an absolute value into two "normal" inequalities.

3. In Chapter I.4, we will learn that polynomial functions like

$$f(x) = 2x^3 + x^2 + 1$$

and rational functions like

$$g(x) = \frac{x^2 + 3x + 2}{x^3 - 1}$$

are *continuous* at each point where they are defined. This just means that

$$\lim_{x \to a} f(x) = f(a)$$

for all a, and that

$$\lim_{x \to a} g(x) = g(a)$$

for every a where $g(a)$ is defined.

Using this, calculate (where defined):
i) $\lim_{x \to 1} f(x)$
ii) $\lim_{x \to 2} g(x)$
iii) $\lim_{x \to 1} g(x)$.

4. Using the same fact about rational functions as in Problem 3), calculate

$$\lim_{x \to a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a}$$

for any nonzero real number a.

[Note: for those who realize this is the derivative of $\frac{1}{x}$ at a, you may *not* simply evaluate the derivative at a. This problem amounts to *showing* that the derivative of this function really is what you think it is.]

5. (Putnam 2002) Given any five distinct points on the surface of a sphere, show that we can find a closed hemisphere which contains at least four of them. (Note: this has *absolutely nothing* to do with our class.)