Math 1A Worksheet 33

December 4th, 2007

1. (Repeat from Worksheet 32) Consider the region bounded by the curves $x + y^2 = 2$ and $x + y = 0$.
   a) Sketch this region, making sure to label the points of intersection.
   b) Think of y as a function of x, and sketch four (tall and narrow) approximating rectangles.
   c) Still thinking of y as a function of x, express the area of this region as a sum of two definite integrals.
   d) Now, thinking of x as a function of y, sketch four (short and wide) approximating rectangles.
   e) Continuing to think of x as a function of y, express the area of this region as a single definite integral.
   f) Evaluate the integrals in c) and e). Did you get the same answer? (You should!)

2. a) Find a number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$, $1 \leq x \leq 4$.
   b) Find a number b such that the line $y = b$ also bisects the same area.

3. For each of the following solids, sketch the solid and find its volume by integration.
   a) A sphere of radius r.
   b) A right circular cone with height h and radius r.
   c) A pyramid with height h and base that is an equilateral triangle with sides a.
   d) The cap of a sphere with radius r. The cap extends from the top of the sphere down to a height h.

4. Again consider the region of problem 1. Find the volume of the solid you get when you rotate this region around: a) The x-axis.
   b) The y-axis.

[Note: in both cases, you have to be careful about what you're doing or you'll double-count some of the volume!]

5. The lines $y = 0$, $x = 1$, and $y = x$ bound a triangular region. Find the volume of the solid you obtain when you rotate this region around the line $y = x$.

6. If a and b are positive real numbers, show that

$$\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx.$$