

Math 1A Worksheet 31

November 28th, 2007

1. a) Suppose

$$y = \int_{\cos x}^{x^2} \cos(u^2) du.$$

Find $\frac{dy}{dx}$.

- b) If f is a continuous function and g and h are differentiable functions, find

$$\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt.$$

2. Let a be a fixed real number. Where is the function

$$g(x) = \int_a^{\sinh x} \cosh^2(t) dt$$

increasing?

3. Let f be a continuous function, and suppose $G(x)$ is an antiderivative for f with $G(2) = 1$. Write $G(x)$ in terms of f , an integral, and a constant.

4. Find

$$\lim_{t \rightarrow 0^+} \int_t^1 t \frac{\cos y}{y} dy.$$

5. Suppose f and g are two integrable functions.

a) Show using Riemann sums that if $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x) dx \geq 0$. [Hint: if every Riemann sum is positive...]

b) Use a) to show that if $f(x) \geq g(x)$ for all $a \leq x \leq b$, then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$.

c) Use b) to show that if $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$. Draw a picture showing why

this is intuitively true.

d) Use b) again to show that $\left| \int_a^b f(x) dx \right| \leq \int_a^b |f(x)| dx$. [Hint: $-|f(x)| \leq f(x) \leq |f(x)|$.] Draw a picture showing why this is intuitively true.

6. Give an example of a function on $[0, 1]$ which is bounded but not integrable. (Note: we haven't really discussed much what "integrable" means. Your example should obviously have no reasonable notion of "area under the graph").