Math 1A Worksheet 29

November 21th, 2007

1. The curves $y = x$ and $y = x^2$ meet in two points and form the boundary of some shape. Find the area of this shape.

2. (Worksheet 28, Problem 4 Revisited) Recall that last time, we showed that
$$\sum_{i=0}^{n-1} ar^i = \frac{a(r^n - 1)}{r - 1}.$$
Let's use this to calculate $\int_0^1 e^x$ using Riemann sums. (We know from the fundamental theorem of calculus that we expect to get $e - 1$, but this is neat to do another way.)
a) Partition $[0, 1]$ into n equal intervals of length $\frac{1}{n}$. Using the *left-hand* endpoint of each interval, write a Riemann sum approximating the above integral.
b) Use the above summation formula to rewrite the Riemann sum as a simpler expression.
c) Now take the limit as $n \to \infty$. You'll need to use l'Hospital's rule.

3. Let's calculate $\int_1^2 \ln x \, dx$. We can't do this directly, since we don't know how to find an antiderivative. Instead, do the following:
a) Draw a graph showing $\int_1^2 \ln x \, dx$ as an area.
b) Reflect this picture across the line $y = x$. What function does $\ln x$ correspond to when reflected in this way? Label as much as you can on the new, reflected picture.
c) Use this new picture to calculate the area.

4. Let
$$f(x) = \begin{cases} \frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0. \end{cases}$$
a) Find $\int_1^2 f(x) \, dx$.
b) Let t be a small real number, greater than 0. Draw a picture that

shows the relationship between $\int_0^1 f(x)\,dx$ and $\int_t^1 f(x)\,dx$, and use this picture to write an inequality relating these two integrals. Find $\lim_{t\to 0^+} \int_t^1 f(x)\,dx$. What does this say about $\int_0^1 f(x)\,dx$?
c) Show the same thing you showed in part b), this time using Riemann sums. [Hint: think about x_1^*.]

5. Have a Happy Thanksgiving! Relax and have fun, but do try to study a little bit ;)