Math 1A Worksheet 28

November 19th, 2007

1. Show geometrically and explain why:
   a) If $f(x)$ is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$.
   b) If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$.

2. Express the following limits as definite integrals. There is more than one way to do each; try to find the simplest possible way.
   a)
   $$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[3\left(1+\frac{2i}{n}\right) - 6\right]$$
   b)
   $$\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}}{n^{3/2}}.$$

3. (Geometric Series):
   a) Let a and r be two real numbers. Write $a + ar + ar^2 + \cdots + ar^{n-1}$ in summation notation. Call this sum $S(n)$.
   b) Assume that $r \neq 1$. Prove that $S(n) = \frac{a(r^n-1)}{r-1}$. [Hint: there are two possible approaches. One is induction, one is algebra.]
   c) Suppose $0 \leq r < 1$. Find $\lim_{n\to\infty} S(n)$.
   d) Only for those who recall series: write the meaning of the above limit in series notation.

4. Let's calculate $\int_0^1 e^x$ using Riemann sums. (We know from the fundamental theorem of calculus that we expect to get $e-1$, but this is neat to do another way.)
   a) Partition $[0,1]$ into n equal intervals of length $\frac{1}{n}$. Using the *left-hand* endpoint of each interval, write a Riemann sum approximating the above integral.
   b) Use the formula proved in part (b) of the previous problem to

rewrite the Riemann sum as a simpler expression.
c) Now take the limit as $n \to \infty$. You'll need to use l'Hospital's rule.

5. (Berkeley Prelim Exam 1990): Suppose $f(x)$ is a continuous function on $[0, 1]$. Show that there is some constant c, $0 \leq c \leq 1$, such that

$$\int_0^1 x^2 f(x)dx = \frac{f(c)}{3}.$$