Math 1A Worksheet 26

November 9th, 2007

1. Consider the function

$$f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ -\sqrt{-x} & x < 0. \end{cases}$$

Show that no matter what initial approximation $x_1 \neq 0$ you pick, Newton's method will fail to find the root of f.

2. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

3. Find all functions $f(x)$ with $f''(x) = e^x$.

4. Suppose that $a_1, a_2, \ldots, a_n$ are real numbers such that

$$a_1 + \frac{a_2}{2} + \frac{a_3}{3} + \cdots + \frac{a_n}{n} = 0.$$

Show that

$$f(x) = a_1 + a_2 x + a_3 x^2 + \cdots + a_n x^{n-1}$$

has a root. [Hint: think about an antiderivative for $f(x)$.]

5. Find the absolute maximum of $(1-x^2)(\cos x)$ for x in $[-100001\pi, 100001\pi]$.

6. (Berkeley Prelim Spring 2001) Suppose $f(x)$ is a continuous function, defined for all real numbers, and suppose moreover that f is periodic with period 1, i.e. $f(x+1) = f(x)$ for all x. Show that $f(x)$ is bounded from above and below (i.e. there exist constants m and M such that $m \leq f(x) \leq M$ for all x), and show that f has an absolute maximum and minimum.