

Math 1A Worksheet 25

November 7th, 2007

1. Story problem time! Sophie Snell, a lifeguard, is sitting at her lifeguard post, 20 meters from the water's edge. She spots a swimmer in trouble; he's floundering 20 meters out into the water, and the point on the beach nearest to him is 40 meters away from the point in the water nearest to Sophie.
 - a) Draw a picture to illustrate the above, or ask me very nicely to draw a picture.
 - b) On land, Sophie can run at 6 meters per second, while in the water, she moves at only 2 meters per second. Given this, discuss where on your picture Sophie should enter the water to reach the swimmer fastest. No equations yet!
 - c) Now, let x be the distance between the point on the water closest to Sophie's lifeguard post and the point where Sophie enters the water. Add x to your picture. Set up an equation for the time it takes Sophie to reach the swimmer in terms of x . Don't solve this yet!
 - d) Take a derivative of the equation you've found and set it equal to 0. Note that there will only be one solution. Now, use the equation you get to find a relationship between the cosines of the angles between the shoreline and each of the two legs of Sophie's path. Use this to make a rough sketch of the shortest path.
 - e) Now solve for x to an accuracy of 1 meter (by method of your choice; calculator allowed).
2. Consider the function

$$f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ -\sqrt{-x} & x < 0. \end{cases}$$

Show that no matter what initial approximation $x_1 \neq 0$ you pick, Newton's method will fail to find the root of f .

3. Find all functions $f(x)$ with $f''(x) = e^x$.
4. (Berkeley Prelim Spring 2001) Suppose $f(x)$ is a continuous function, defined for all real numbers, and suppose moreover that f is periodic with period 1, i.e. $f(x+1) = f(x)$ for all x . Show that $f(x)$ is bounded from above and below (i.e. there exist constants m and M such that $m \leq f(x) \leq M$ for all x), and show that f has an absolute maximum and minimum.