Math 1A Worksheet 24

November 4th, 2007

1. Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x} - \frac{x}{|x|}$. Show that

$$\frac{d}{dx}(f(x) - g(x)) = 0,$$

but that $f(x) - g(x)$ is *not* constant. Explain why this isn't a contradiction.

2. Show that:
a) $e^x \geq 1 + x$ for all $x \geq 0$, and
b) $e^x \geq 1 + x + \frac{x^2}{2}$ for all $x \geq 0$.

3. Sketch a graph of a function satisfying all of the given conditions: $f'(-1) = f'(1) = 0$; $f'(x) < 0$ if $|x| < 1$; $f'(x) > 0$ if $|x| > 1$; $f(-1) = 4$, $f(1) = 0$; $f''(x) < 0$ if $x < 0$; $f''(x) > 0$ if $x > 0$.

4. Suppose that both $f(x)$ and $g(x)$ are twice-differentiable and concave-up for all x. What condition on f will ensure that $f(g(x))$ is concave-up for all x?

5. Sketch a graph of a function satisfying all of the given conditions: $\lim_{x\to 3} f(x) = -\infty$; $f''(x) < 0$ if $x \neq 3$; $f'(0) = 0$; $f'(x) > 0$ if $x < 0$ or $x > 3$; $f'(x) < 0$ if $0 < x < 3$.

6. Calculate $e^{\sinh^{-1}(1/2)}$. This number is called the *golden ratio*, often denoted by the greek letter ϕ. Show that

$$\phi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \cdots}}}.$$