Math 1A Worksheet 23

October 31st, 2007

1. a) Give an example of a differentiable function f and a point c such that $f(c)$ is a local maximum and $f''(c) = 0$. Can you use the second derivative test in this situation?
   b) Give an example of a function f and a point c such that $f(c)$ is a local minimum and $f''(c)$ does not exist. Can you use the second derivative test in this situation?
   c) Challenge: find a function as in b) where f is also differentiable everywhere.

2. Consider the quadratic polynomial $f(x) = ax^2 + bx + c$ where a, b, and c are some real numbers and $a \neq 0$. Show that f has exactly one critical point and no inflection points. When is f concave up? When is it concave down?

3. For each of the following functions, find: (1) the horizontal and vertical asymptotes, (2) the intervals of increase/decrease, (3) the local extrema, (4) the intervals of concavity and the inflection points, and (5) use (1)-(4) to sketch a graph.
   a) $f(x) = x^4 - 6x^2$,
   b) $f(x) = \ln(1 + x^2)$,
   c) $f(x) = \sin^{-1}(\tanh(x))$.

4. Find
$$\lim_{x \to 0} \frac{(x^2)^{x^2} - 1}{x}.$$

5. Let $f(x)$ be a differentiable function defined for all real numbers. Recall that a *fixed point* of f is a number a such that $f(a) = a$. Suppose that for every x, we have $f'(x) \neq 1$. Show that f has at most one fixed point. [Hint: use an auxillary function whose zeros correspond to fixed points of f. We did something similar to this before.]

6. (Practice with polynomials.) Let $f(x)$ be a quadratic polynomial as in Problem 2. Suppose f has two real zeros r and s. Show that $f'(r) + f'(s) = 0$. Show also that the unique critical point of f occurs halfway between r and s.