

Math 1A Worksheet 22

October 29th, 2007

1. By taking derivatives, show that $\sin^{-1}(\tanh x) = \tan^{-1}(\sinh x)$. [Note: remember, to show $f = g$, it is NOT enough to show $f' = g'$!]
2. Using the MVT, show that $\sqrt{1+x} < 1 + \frac{1}{2}x$ for $x > 0$. [Hint: consider the function $f(x) = 1 + \frac{1}{2}x - \sqrt{1+x}$.]
3. Show that $x^{1001} + 10x + 1$ has exactly one real root.
4. Use Rolle's theorem to show that a polynomial of degree 3 has at most 3 real roots.
5. Along the same lines: suppose that f is twice differentiable on all of \mathbb{R} and has three real roots. Show that f'' has at least one real root.
6. Show that $|\sin a - \sin b| \leq |a - b|$ for all a and b .
7. (The Ultimate l'Hospital's Rule Problem): Find

$$\lim_{x \rightarrow 0} \frac{e^{-1/x^2}}{e^{-1/\sin x}}.$$