

Math 1A Worksheet 21

October 24th, 2007

1. If f and g are two differentiable functions, find $\frac{d}{dx} f(x)^{g(x)}$.
2. For each of the following limits, state whether the expression is an indeterminate form, and if so, what the indeterminate form is. Then find the limit. [Note: keep in mind when doing these limits that l'Hospital's rule, while useful, is no substitute for actually using your brain.]

a)

$$\lim_{x \rightarrow \pi^-} \frac{\sin x}{1 - \cos x}.$$

b)

$$\lim_{x \rightarrow 0^+} x^{1/x^2}.$$

c)

$$\lim_{x \rightarrow \infty} \ln(x) - x.$$

d)

$$\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right).$$

e)

$$\lim_{x \rightarrow 0} \frac{\tan x}{\tanh x}.$$

3. True or false? (If false, give a counterexample):
 - a) If f has a local extremum at c , then c is a critical number of f . [Note: "extremum" means "maximum or minimum."]
 - b) If c is a critical number of f , then f has a local extremum at c .
 - c) Every function has a local extremum.

4. Prove the identity: $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.
5. Prove or disprove: there are infinitely many prime numbers p such that $p + 2$ is also a prime number. [Note: this may be slightly tricky.]