

Math 1A Worksheet 20

October 22nd, 2007

1. Find

$$\lim_{x \rightarrow 0^+} \frac{\cosh x}{\sinh x}.$$

2. The most important thing about l'Hospital's Rule is to know when you can and cannot apply it.

a) Find $\lim_{x \rightarrow 0} \frac{x}{x+1}$. Did you need to use l'Hospital's Rule?

b) What value would you have gotten if you had tried to use l'Hospital's Rule? Why does l'Hospital's Rule not apply in this situation?

3. Give examples of indeterminate forms of each of the following types, and try to solve each of your examples, using l'Hospital's Rule or otherwise:

- a) $\frac{0}{0}$,
- b) $\frac{\infty}{\infty}$,
- c) 1^∞
- d) 0^0 .

4. Let f be a twice-differentiable function such that f'' is continuous. Use l'Hospital's rule to find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

[Note: be *very* careful about which variable you are differentiating with respect to!]

5. (More about differential equations):

a) We have seen one differential equation that we know something about: $y' = ky$. Demonstrate that $y = Ae^{kx}$ is a solution to this equation.

b) Find a solution to the differential equation $y'' = -y$. There are two solutions that we know very well! Show that if f and g are two

solutions to this equation, then $f + g$ is also a solution.

c) Now consider the differential equation $y'' = k^2y$. Then $y = Ae^{kx}$ is still a solution to this equation. Find another solution.