Math 1A Worksheet 2

August 29th, 2007

1. Is there a function all of whose values are equal to each other? If so, graph your answer, if not, explain why.

2. True or False. If true, try to give a proof. If false, give a counterexample.
   i) For all real numbers a and b, we have $|a+b| \leq |a| + |b|$.
   ii) For all functions f and g, we have $|f(x)+g(x)| \leq |f(x)| + |g(x)|$.

3. Consider the polynomial functions

   $$f(x) = -x^2 + 1, \quad g(x) = (x-1)^2, \quad h(x) = x^3.$$

   i) Find all x such that $f(x) \leq 2$. Do the same for $g(x)$ and $h(x)$.

   ii) Find all x such that $|f(x)| < 2$. Do the same for $|g(x)|$ and $|h(x)|$.

   iii) Can you find an *upper bound* for f? That is, can you find a number M such that $f(x) \leq M$ for all x? What about g and h?

   iv) Can you find a *lower bound* for f? I leave it to you to define this. What about g and h?

   v) What about finding upper bounds for f *restricted* to $[-1,1]$? That is, can you find M such that $f(x) \leq M$ for all x in $[-1,1]$? How about g and h? Can you find lower bounds for f, g, and h on $[-1,1]$?

4. Let f and g be two functions. Prove or give a counterexample: If M is an upper bound for $|f(x)|$ and N is an upper bound for $|g(x)|$, then

   $$|f(x)+g(x)| \leq M + N$$

   for all x. *Hint: use question 2*!

5. Last time, we discussed the function $f(x) = \frac{|x|}{x}$. Draw the graph of this function. What are

$$\lim_{x \to 0^-} f(x) \quad \text{and} \quad \lim_{x \to 0^+} f(x)?$$

What does this tell us about $\lim_{x \to 0} f(x)$?