

Math 1A Worksheet 19

October 19th, 2007

1. Find the following limits:

a)

$$\lim_{x \rightarrow \infty} \sin^{-1}(\tanh(x))$$

b)

$$\lim_{x \rightarrow \infty} \cosh(x) - \sinh(x)$$

2. Find, with proof, the following derivatives:

a) $\frac{d}{dx} \cosh x$,

b) $\frac{d}{dx} \cosh^{-1}(x)$.

3. Prove that $\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$ using the following steps:

a) Show that $\cosh(\sinh^{-1}(x)) = \sqrt{1 + x^2}$.

b) Prove the formula $\cosh x + \sinh x = e^x$.

c) Substitute $\sinh^{-1}(x)$ for x in equation b) and simplify.

4. (More about differential equations):

a) We have seen one differential equation that we know something about: $y' = ky$. Demonstrate that $y = Ae^{kx}$ is a solution to this equation.

b) Find a solution to the differential equation $y'' = -y$. There are two solutions that we know very well! Show that if f and g are two solutions to this equation, then $f + g$ is also a solution.

c) Now consider the differential equation $y'' = k^2y$. Then $y = Ae^{kx}$ is still a solution to this equation. Find another solution.