

Math 1A Worksheet 18

October 17th, 2007

1. Find the following limits:

a)

$$\lim_{x \rightarrow -1^+} \sin^{-1}(x).$$

b)

$$\lim_{x \rightarrow 4\pi^-} \cos^{-1}(\cos x).$$

2. Sketch graphs of $f(x) = \sin(\sin^{-1} x)$ and $g(x) = \sin^{-1}(\sin x)$. Find $g'(x)$.
3. The graphs of $\sin^{-1}(x)$ and $\cos^{-1}(x)$ intersect in a single point. Find that point.
4. A freshly roasted turkey (a piping-hot 70 degrees Celsius!) floats in the vacuum of space (a frigid -270 degrees C!). Just ten minutes later, the turkey has reached a frigid -20 degrees C. Find an equation describing the turkey's temperature, and use it to find the initial rate of cooling of the turkey. Avoid asking existential questions.
5. You travel back in time to the year 0 AD (remember what we just said about existential questions!), where you invest a single penny at a continuously compounded interest rate of just 1 percent. You return to the present time, track down the banker's (great)⁶⁰-grandnephew, and collect... how much?

[Note: also ignore such issues as “who would accept pennies seventeen hundred years before the founding of America?” or “what sort of banker keeps tabs for 2000 years?”]

6. Let

$$f(x) = \begin{cases} x \arctan(1/x) & , \quad x \neq 0 \\ 0 & , \quad x = 0. \end{cases}$$

Is f continuous at 0? Is f differentiable at 0?

7. Suppose a_1, a_2, \dots, a_{14} are some 14 integers. Show that

$$\prod_{1 \leq i < j \leq 14} (a_i - a_j)$$

is divisible by $2002 = 2 \cdot 7 \cdot 11 \cdot 13$.