

Math 1A Worksheet 17

October 10th, 2007

1. Give an example of two functions f and g , defined for all real numbers, such that neither f nor g is one-to-one, but $f \cdot g$ is one-to-one.
2. Give an example of two functions f and g , defined for all real numbers, such that $f \circ g$ is one-to-one but f is not one-to-one. [Note: g may (and indeed, must!) be one-to-one!]
3. By drawing appropriate triangles and thinking hard, find the following:
 - a) $\sin(\cos^{-1} x)$
 - b) $\cos^{-1}(\sin x)$.
4. Let $y = \cos^{-1}(x)$. Use implicit differentiation and the formulas $\cos y = x$ and $\cos^2 + \sin^2 = 1$ to find a formula for $\frac{d}{dx} \cos^{-1}(x)$ in terms of x alone.
5. Find

$$\lim_{x \rightarrow \infty} \arctan \left(\frac{1}{\cos \left(\arcsin \left(\frac{\sqrt{x^2-1}}{x} \right) \right)} \right).$$