# Math 1A Worksheet 15

October 8th, 2007

1. Find
$$\lim_{x \to \pi} \frac{e^{\sin x} - 1}{x - \pi}.$$

2. Let $f(x) = 8x^3 + 2x + 2 + \cos x$. Find $f^{-1}(3)$ and $f^{-1}(-\pi^3 - \pi + 2)$.

3. Let $g(x) = x + \ln x$. For which $x$ do we have $-1 < g^{-1}(x) \leq 3$? (Be careful; this is an easy one to get backwards, and the whole point of this problem is to think hard about inverse functions!)

4. (A simple example of logarithmic differentiation.) If $y = (x+2)^5$, we obviously know how to find $y'$ using the power rule and chain rule. For practice, let's do this by logarithmic differentiation instead:
   a) Take ln of both sides of this equation, and differentiate with respect to $x$.
   b) Now do algebra to get an equation for $y'$ in terms of $x$. Does this look like the usual formula for the derivative of $(x+2)^5$?
   c) Now, we almost certainly cheated twice in this computation, and these two cheats cancelled out each others' effects. Explain why the above calculation is actually only valid for $x > -2$.
   d) We can fix the problem in c) by instead taking $\ln|y| = \ln(|x+2|^5)$ and going through steps a) and b). This still fails to give us information about one point. Which one?

5. Oftentimes a function is *not* one-to-one, but we can restrict the domain of the function to make it one-to-one and define an inverse to this restricted function. We have seen an example: $\sqrt{x}$ is the inverse of $x^2$ *restricted to* $[0, \infty)$. Let's look at another good example:
   a) The function $\sin x$ is *not* one-to-one. How can we restrict its domain to get a function which is one-to-one?
   b) Explain how we use part a) to define the well-known function $\sin^{-1}(x)$.