Math 1A Worksheet 13

October 1st, 2007

1. A fixed quantity of a gas kept at constant temperature in a closed container obeys a law of the form $PV = C$, where P is the ambient pressure and V is the volume the gas occupies. Suppose we increase the pressure of a system of this form at a constant rate. Explain why knowing only this rate would **not** give us enough information to find the rate of change of the volume. Do this using both calculus and real-world reasoning.

2. Suppose the figure below is a regular hexagon with side length 3 inches. Find the area of the shaded region in terms of h.

   Suppose now that this hexagon is actually a cross-section of a hexagonal fish tank which is 3 inches wide. We begin filling this fish tank with water at a rate of 1 cubic inch per second. How quickly is the water level rising when the water has reached a depth of $(3\sqrt{3})/4$ inches?

3. (Quadratic Approximation) Let f be a function which is twice-differentiable at a point a. Just as we can approximate $f(x)$ near a by a line (i.e. a degree-1 polynomial), we can also approximate it by a quadratic function (i.e. a degree-2 polynomial). We find this approximation as follows: write $P(x) = A + B(x-a) + C(x-a)^2$. We want to have $P(a) = f(a)$, $P'(a) = f'(a)$, and $P''(a) = f''(a)$. Find a formula for A, B, and C in terms of $f(a)$, $f'(a)$, and $f''(a)$.

4. Apply the previous problem to find a quadratic approximation for $f(x) = \sin x$ near $a = \pi/2$. Graph $f(x)$, the linear approximation to $f(x)$ at a, and the quadratic approximation to $f(x)$ at a all on the same graph.

5. Is it possible to place 8 queens on a chessboard in such a way that no queen can capture any other?