

Math 1A Worksheet 11

September 26th, 2007

1. Find the following limits:

a)

$$\lim_{x \rightarrow 0} \frac{\sin(\sin x)}{\sin x},$$

b)

$$\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right),$$

c)

$$\lim_{x \rightarrow \infty} \left(x - x \cos\left(\frac{1}{x}\right)\right).$$

2. Consider the curve defined by $y^3 + xy + x^4 = 11$. Find the slope of the tangent to this curve at $(1, 2)$. Find a formula for $\frac{d^2y}{dx^2}$ in terms of only x and y .

3. Find the following derivatives by finding a pattern:

a) $\frac{d^{99}}{dx^{99}} \sin x,$

b) $\frac{d^{100}}{dx^{100}} \cos x,$

c) $\frac{d^{50}}{dx^{50}} \cos 2x,$

d) $\frac{d^{35}}{dx^{35}} x \sin x.$

4. Suppose f is a differentiable function with f' continuous, and suppose moreover that $f(0) > f(1)$ and $f'(1) > 0$. Show that there is some point at which f has a horizontal tangent vector. [Hint: what does $f(0) > f(1)$ tell us about f' ?]

5. What is the maximum number of bishops one can place on an 8×8 -chessboard such that no bishop can capture any other?