

Math 1A Worksheet 10

September 24th, 2007

1. In preparation for the chain rule, practice with composition. If $k(x) = f(g(h(x)))$, complete the following table:

If $f(x) = x^2$,	$g(x) = \sin(x)$,	$h(x) = \cos(4x)$,	then $k(x) = \underline{\hspace{2cm}}$.
If $f(x) = \frac{1}{x}$,	$g(x) = \underline{\hspace{2cm}}$,	$h(x) = (12 - x^2)$,	then $k(x) = (12 - x^2)^{-2}$.
If $f(x) = \underline{\hspace{2cm}}$,	$g(x) = x$,	$h(x) = \underline{\hspace{2cm}}$,	then $k(x) = \cos(\tan x)$.
If $f(x) = \underline{\hspace{2cm}}$,	$g(x) = \underline{\hspace{2cm}}$,	$h(x) = \underline{\hspace{2cm}}$,	then $k(x) = [1 - (3x - 2)^3]^4$.

2. Let

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}.$$

Find $f'(x)$. Where is $f'(x)$ continuous?

3. Use the chain rule to show that the derivative of an even function is an odd function and the derivative of an odd function is an even function. Suppose $g(x)$ is a differentiable even function. Find $g'(0)$. Suppose $h(x)$ is an odd function. Must it be true that $h'(0) = 0$?
4. Let $g(x) = |\sin x|$. At what points is $g(x)$ differentiable? Give a formula for $g'(x)$ and sketch its graph. [Hint for finding a nice formula: $|x| = \sqrt{x^2}$.]
5. Let n be a positive integer. Find

$$\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1}.$$

6. Find all prime numbers p such that $p + 2$ and $p + 4$ are also prime.