

Math 1A Practice Final
December 7th, 2007

Name _____ SID _____

1. Answer the following questions True or False. Use T for true and 0 for false. If false, give a counterexample.

___ a) Every continuous function is differentiable.

Counterexample (if false):

___ b) The product of two everywhere-differentiable strictly increasing functions is strictly increasing.

Counterexample (if false):

___ c) Every continuous function has an antiderivative.

Counterexample (if false):

___ d) Every integrable function is continuous.

Counterexample (if false):

___ e) If f and g are two functions with the same antiderivative, then $f = g$.

Counterexample (if false):

2. Show that the function $f(x) = e^x - x - 2$ has exactly two roots.

3. Evaluate the following limits:

a) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{8}{n} \sin\left(1 + \frac{2i}{n}\right)$.

b) $\lim_{n \rightarrow \infty} \frac{\sqrt{3n^4 + n^3 + \sin n}}{n^2 + 2n - 1}$.

c) $\lim_{x \rightarrow 0} \frac{\arccos(x) - 1}{\arcsin(x)}$.

4. Simplify/evaluate the following expressions:

a) $\int \frac{1+x}{\sqrt{1+x^2}} dx$.

b) $\frac{d^2}{dx^2} \int_0^{x^2} \left(\int_0^t \sqrt{1+u^5} du \right) dt$. [Note: here “simplify” means “get rid of the derivative.” Your answer will still have an integral in it.]

c) $\int_0^{\sqrt{3}} \sqrt{4-x^2} dx$.

d) $\int_0^{\pi/2} (\sin^3 x \cos x) e^{\cos^2 x} dx$. [Hint: $ue^u - e^u$ is an antiderivative for ue^u .]

5. Use the “shell method” to find the volume of the solid generated by rotating the region bounded by $y = \ln x$, $y = 0$, $y = 1$, and $x = 0$ around the x -axis. [Hint: $te^t - e^t$ is an antiderivative for te^t].