

Math 113 Midterm Exam  
July 16th, 2009

Name \_\_\_\_\_

Question	Score	Possible
1.1		2
1.2		3
1.3		4
1.4		3
2.1		3
2.2		5
2.3		4
$\Sigma$		24

## 1 Computations

**Problem 1.1.** (2 points) Let  $\sigma \in S_7$  be the element  $\sigma = (1235)(237)(45)$ . Write  $\sigma$  as a product of disjoint cycles, and compute the order of  $\sigma$ .

**Problem 1.2.** (3 points) Find all possible cycle types of permutations in  $S_6$ . For each cycle type, state whether permutations of that cycle type are even or odd. Circle the cycle types which correspond to elements of  $A_6$ .

**Problem 1.3.** Let  $G = (\mathbb{Z}/11\mathbb{Z})^\times$ . Do the following:

1. (1 point) Show that  $G$  is cyclic by finding a generator.
2. (3 points) List all subgroups of  $G$ , and for each subgroup of  $G$ , give a generator for that subgroup.

**Problem 1.4.** (3 points) Show that  $\mathbb{Z}/6\mathbb{Z} \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$  by writing down an explicit isomorphism  $f : \mathbb{Z}/6\mathbb{Z} \rightarrow \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$ . (You must prove that your function  $f$  is in fact an isomorphism).

## 2 Theory

**Problem 2.1.** (3 points) Show that  $D_n$  has a subgroup of order  $k$  for every  $k$  dividing  $n$ .

**Problem 2.2.** Let  $G$  be an abelian group, and let  $g, h \in G$  be elements.

1. (2 points) Let  $H$  be the set  $\{g^n h^m \mid n, m \in \mathbb{Z}\}$ . Show that  $H$  is a subgroup of  $G$ .
2. (3 points) Suppose that there exists some element  $a \in G$  and integers  $k, \ell \in \mathbb{Z}$  such that  $g = a^k$  and  $h = a^\ell$ . Show that the subgroup  $H$  defined in part (1) is cyclic.

**Problem 2.3.** (4 points) State and prove Lagrange's Theorem.

