

# MANY CHEERFUL FACTS

presents

## Generalized Fermat Equations and Descent: A Love Story

a talk by David Brown

1:10 pm - 2:00 on Wednesday, December 7th, in room  
891.

Curves are what get me up in the morning and what keep me cheerful all afternoon. I'm pretty fond of surfaces too, particularly the surfaces

$$Ax^\alpha + By^\beta + Cz^\gamma = 0. \tag{1}$$

These surfaces usually have only finitely many rational points. In particular, there is the surprising result that this surface has finitely many rational points if  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} - 1 < 0$ , and infinitely many if  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} - 1 > 0$ .

I will first motivate this result with a simple and interesting (but also completely rubbish) heuristic. Then I will begin talking about descent; first I will work out a classical problem by elementary means, and then describe what happened from a modern viewpoint. Finally, I will explain descent and use it to solve 1. Along the way I should have a chance to mention many of the surprising theorems relating geometry to number theory that attract me to this subject.

*I am the very model of a modern Major General,  
I've information vegetable, animal, and mineral,  
I know the kings of England, and I quote the fights historical  
From Marathon to Waterloo, in order categorical;  
I'm very well acquainted, too, with matters mathematical,  
I understand equations, both the simple and quadratical,  
About binomial theorem I'm teeming with a lot o' news,  
With many cheerful facts about the square of the hypotenuse!*

- Gilbert & Sullivan *P o P*