Math 74 – Homework Assignments.

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Spring 2008.

1 Due 2/1.

From the book do: (1.1) 3, 4, 6, (1.2) 2, 4, 6, 8, 10, 12, 15, (1.3) 2, 4, 6, 8.

Also: Find two substantially different proofs of Pythagorus' theorem and present them (acknowledging your sources). Each of these will probably use some other theorems from High School Geometry; you should state these clearly, but need not prove them.

2 Due 2/8.

From the book do: (1.4) 2, 6, 7, 9, 14, (1.5) 3, 5, 7(b), (2.1) 2, 3, 5, 6, 8

Also: Read the handout[1] on axioms for the real numbers and use it to prove the following facts directly from the axioms:

1. If $ab = ac$ and $a \neq 0$, then $b = c$;
2. If $ab = 0$ then $a = 0$ or $b = 0$.

[There are ways of proving both these things which are 'structurally'[2] very simple and should remind you of proving algebraic or trigonometric identities, or doing the logic proofs using the laws from last week. This style of proof is, in a way, the bread and butter – we'll add more sophisticated structures to it as we go through this course.]

3 Due 2/15.

From the book do: (2.2) 2, 3, 7, 9, 10 (2.3) 2, 3, 5, 6, 10, 14.

No reading assignment this week, to give you time to start preparing for the midterm.

4 Due 2/23.

From the book do: (3.1) 2, 3, 7, 8, 12, 15.

Also: Referring to p. 84 of M. E. Munroe, *Introduction to Measure and Integration*, Reading, MA: Addison-Wesley (1953), answer the following questions.

[1]The handout should also be available on http:// www.maths.ox.ac.uk/ current-students/ undergraduates/ lecture-material/ Mods/ analysis1/ in the documents *Axioms for the real numbers* and the first four pages of *Week 2*, but they may update the site and change this at some point.

[2]See the preface to Velleman for more details about this sense of *structurally*.

1. For the three corollaries listed on this page, write down the hypotheses and conclusions and use the rules we've learnt so far to transform each of these into givens and goals where the goal is just a single statement. (Do some using Direct Proof (p. 90 of Velleman) and some using Proof by Contrapositive (p. 91)).

2. Can you tell from the proofs given whether Munroe uses Direct Proof or Proof by Contrapositive here?

5 Due Leap Day.

From the book do: (3.2) 2, 3, 6, 9, 12 (3.3) 2, 6, 13, 18, 21, 23.

Also: This question relates to p.176 of A. Hatcher, *Algebraic Topology*, Cambridge: CUP (2001). [You're not meant to be solving the exercises on the handout.]

1. Analyze the logical form of Corollary 2B.7.

2. Given its logical form, how would expect it to be proved (make reference to section 3.3 of Velleman). Write the top and bottom of a proof of it. (You will, of course, not be able to fill in the middle).

3. Now, look at the proof Hatcher gives. Does it follow this form? Does it seem to be using another technique?

6 Due 3/8.

From the book do: (3.4) 2, 5, 8, 10, 13, 20, 22, 26 (3.5) 2, 6, 9, 13, 21, 24, 31.

Also: This question refers to pp. 171-2 of W. Hodges, *A shorter Model Theory*, Cambrdige: CUP (1997).

1. Theorem 6.3.1 claims that, under certain hypotheses, 7 different statements are equivalent. If all the possible conditional statements were proved, how many would that be? What's the smallest number that could be gotten away with? How many are, in fact, proved?

2. Check that the conditionals proved are in fact sufficient to show that all seven statements are equivalent, by drawing a directed graph[3] whose nodes are the statements (a) through (g) and whose arrows are the conditionals which are proved. How can you deduce from Hodges' proof that (a) implies (d), for example?

3. Find one conditional which is proved by direct proof and one which is proved by contrapositive.

7 Due Π-day.

From the book do: (3.6) 2, 5, 7, 9(a-c), 12; (4.1) 3(b,c), 10.

Also: This question relates to p. 111 of S. MacLane, *Categories for the Working Mathematician*, New York: Springer-Verlag (1971).

1. The proof of Theorem 2 doesn't have a box or other typographical feature to tell you where it ends. Where should the box be?

[3] If you don't know what a directed graph is, here's an opportunity to develop your research skills!

2. Ignoring the *moreover* statement at the end of the statement of the theorem, verify that the theorem can be analysed as being in the form $P \to (\exists! x)Q(x)$.
3. Which proof technique (direct proof or contrapositive) is used to deal with the fact that the goal is a conditional?
4. Which way of proving unique existence is used?

8 Due 4/4.

From the book do: (4.2) 2, 5, 8(a), (4.3) 2, 12, 15, 22, (4.4) 2 (a,b), 3, 6, 12 , 23.

Also: tba.

9 Due 4/11.

From the book do: (4.6) 4, 8, 14, 17, 24, 25, (5.1) 8, 11, 14, 15.

Also: Find an equivalence relation 'in the wild'. That is, find a math book in which a relation is defined and it is stated (and possibly proved) that it is an equivalence relation. You should turn in a photocopy of the definition of the relation and the statement that it's an equivalence relation together with a proper bibliographic reference to the book. The bibliographic style I've been using in this document is one option, but so is CMoS, MLA, etc.

10 Due 4/18.

From the book do: (5.2) 6, 13, 15, 19, (5.3) 6, 17, 18, (5.4) 1, 2 (6.1) 2, 9(b), 15, 17

Also: Find an example of a proof by induction 'in the wild'. Follow the instructions given in the homework that was due 11/9, and also mark on your photocopy where the base case and where the inductive step are.

11 Due 4/25.

From the book do: (6.2) 5, 7, 12, 16, (6.3) 3, 10, 13, 19.

12 Due 5/2.

From the book do: (6.4) 4, 13, 16, 19, (7.1) 2(a), 3, 10, 21 (a,b).

13 Due 5/9.

From the book do: (7.2) 5, 10, 11 (7.3) 2, 3, 11.

14 Due Never.

These exercises are optional, but I will be posting solutions if you want to learn the material I'm lecturing on in the last week.

(4.5) 2, 4, 17, (6.5) 4, 7, 9.