Math 53: Writing Tasks.

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For due dates for these tasks, please see the calendar on my webpage. All these tasks should be written in your own hand (unless you have a relevant accommodation from DSP). They should also be written up not as exercises but as a report: what you turn in should make sense to some-one who has not seen the question. I will give more guidance on this in class.

1 Integration review.

I would like you to write a review of integration techniques. The target audience for this piece of writing is someone who studied techniques of integration a few years ago, but needs a refresher for a multivariate calculus class they're about to take.

Length. You should limit yourself to four sides of letter paper, though three should be sufficient. If you feel you have unusually large handwriting and wish to use more you can come and see me.

Subject matter. Your review should cover basic substitution, integration by parts and a guide to trig integrals. Though you should not cover these examples, someone who's read your review should be able to tackle the following integrals:

$$\int_0^1 x\sqrt{x^2+1}\,dx$$

$$\int x\sin(x)\,dx$$

$$\int e^x\cos(x)\,dx$$

$$\int \arctan(x)\,dx$$

$$\int \sin^4(x)\,dx$$

$$\int \sin^3(x)cos^4(x)\,dx.$$

Technique. You may wish to include some of the following: worked examples, sketched examples, strategy, step-by-step instructions, exercises, tables, references.

Assessment. You will be assessed on four things:

1. Accuracy. Is what you've written mathematically accurate?
2. Relevance. Is what you've written relevant to the task?
3. Coverage. How much of what was asked for did you cover?
4. Presentation. Have you written in cogent prose? Is your notation helpful to the reader? Is everything clear?

2 Rotated Conics.

In this writing task, you will investigate the graphs of equations of the forms $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$. In class we discussed the case $B=0$ and saw that we either got a conic, or point or the empty set. This is still true when $B\neq 0$. How much you have to do for this task depends on what grade you want for it, as follows:

⊕ For a C; you should produce plots of the following equations (showing all their relevant features) and classify the graphs as ellipses, parabolæ and hyperbolæ. (Remember that a circle is a type of ellipse):

$$x^2+y^2=16 \tag{1}$$
$$x^2+xy+y^2=16 \tag{2}$$
$$x^2+4xy+y^2=16 \tag{3}$$
$$x^2+y^2-8x-10y+16=0 \tag{4}$$
$$x^2+xy+y^2-8x-10y+16=0 \tag{5}$$
$$x^2+2xy+y^2-8x-10y+16=0 \tag{6}$$
$$x^2+4xy+y^2-8x-10y+16=0 \tag{7}$$

You should also provide me with a description of how you made the plots that would allow me to produce them if I had never used the software you used before (see below for some suggestions). If your software is sophisticated enough to allow you to enter the above equations directly, also explain how you would have coped if your software had only allowed you input equations in the form $y=f(x)$.

⊕ For a B; you should (do the above and) calculate the discriminant B^2-4AC for each of the above equations and make a conjecture about how one can determine whether the graph will be an ellipse, parabola or hyperbola based on the discriminant. You should test your conjecture by writing down three new equations, one of which you think will be an ellipse, one a parabola and one a hyperbola and then graphing them.

⊕ For an A; you should (do the above and) give me a proof that your conjecture is true. [Hint: to get an equation for a rotated conic from one for a regular conic, let $\mathbf{u}$ and $\mathbf{v}$ be perpendicular 2D unit vectors and replace x and y in the equation by $\mathbf{u}\cdot\langle x,y\rangle$ and $\mathbf{v}\cdot\langle x,y\rangle$ respectively. For full credit, you need to explain how you're using this hint in your proof, and not just give me long algebraic computations (though you will probably need to give me long(ish) algebraic calculations)].

How much of the above you do will basically determine your grade, with the opportunity to gain / suffer a plus or minus on it for accuracy, style and presentation. Ideally, you would include printouts of the graphs you produce (as a separate sheet, or glued into your work), but if you produce them on a graphing calculator and can't transfer them onto a computer you can make neat copies of them by hand.

If you have access to a Mac with OS X, then this will have come 0bundled with a program called Grapher (in Finder, go to Applications then to Utilities and scroll down till you find it). In this program, you can enter the equations directly and see plots. There are variety of programs available for Windows that will do this, including free ones on-line. This site (http://www.uncwil.edu/courses/mat111hb/demo/jcalc.html) will plot equations in which y is the subject and doesn't require downloading, but I'm sure you can find something much better given a bit of time Googling.

What you should turn in to me should be a clearly written, well explained report; I don't need to see your rough work. As for all these assignments, you should do them alone, but you're encouraged to do peer review. If you do do peer review, write a note on your work saying who read and commented on your work (I give credit for this).

3 Standing waves.

For this task, you are encouraged to read Chapter 7 (especially sections 1 and 2, but 3 is interesting too, if you skip some of the techincal deatils[1]) of *From Calculus to Chaos* by David Acheson. I've placed two copies of the book on reserve at Moffitt, which means you can only borrow them for a maximum of two hours, so everyone should get a chance to look over the next week. Just in case there is some complication, though, here is the question I would like you to answer, which is exercise 7.2 in the book:

> Suppose that a stretched string is fixed at two end-points $x = 0$ and $x = l$, say, so that $z = 0$ there for all time t. Investigate the natural modes of vibration of the string by seeking solutions to the wave equation
>
> $$T\frac{\partial^2 z}{\partial x^2} = \rho\frac{\partial^2 z}{\partial t^2}$$
>
> of the form
>
> $$z = f(x)\sin(\omega t)$$
>
> and deducing that
>
> $$T\frac{d^2 f}{dx^2} + \rho\omega^2 f = 0.$$
>
> Solve this equation, subject to the boundary conditions at $x = 0$ and $x = l$, and show that such vibrations can occur only at certain *natural frequencies*
>
> $$\frac{\omega}{2\pi} = \frac{N}{2l}\sqrt{\frac{T}{\rho}} \qquad N = 1, 2, 3, \ldots$$
>
> Show too that the higher the value of N the larger the number of *nodes*, ie. points of zero displacement on the string – a result well known to musicians.[2]

What you hand in should include around a half page of discussion of the wave equation before leaping into doing calculations. In particular, you should give interpretations for all the symbols. The math should be written up as part of a continuously flowing piece of paragraphed prose that would make sense to someone who hadn't read the question, let alone the chapter of the book.

Some extra credit will be available for including a brief (up to half a page) discussion of why the result is "well known to musicians" [hint: Acheson plays electric guitar], however it will be possible to attain the highest grade without doing this. As always on these assignments, if you engage in peer review, you must record this on your work (there is extra credit available for doing this).

[1] Actually, the whole book is pretty interesting, and I'm only slightly biased due to being a former student of the author.

[2] pp. 105-6, op cit.

4 Designing a Dumpster.

The briefing for this writing-task can be found on p. 963 of Stewart. Write this up as you would a report to someone who'd asked you to make recommendations on how to optimize their dumpster design (although you should probably include more details in your calculations than you would in such a report), rather than like a collection of four separate homework problems. If you like, you can include a digital photo of your dumpster, though you don't have to.

By the way, there's a dumpster by the South East corner of Evans Hall if you can't find another one.

5 Generalizing Newton's Method.

Please do Problems Plus 6 on page 978-9 of the book. As usual, the presentation should make sense to someone who hasn't read the question so explain what you're doing and why you're doing it.

6 Probability.

I'll give you a question off an old Oxford exam[3] – question 2 from the exam linked here: http://tinyurl.com/g6tv3

Here are some hints. First of all you'll need to know that the p.d.f. of X and Y is $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$. (This is the *standard normal distribution*). For (a), recall that

$$f_{(U,V)}(u,v) = \frac{\partial(u,v)}{\partial(x,y)} f_{(X,Y)}(x,y).$$

For (b), there probably is a calculation you can do to work this out, but there's a much easier informal argument if you understand what's going on. For (c), *joint pdf* is what I've been calling *jdf*. For (d), my only hint is to think before you plow into calculations (you don't need to do much).

[3] I took the incarnation of this exam a year before this one. The one I took didn't have a question that was suitable for you unfortunately, as it would have been kind of cute to assign one of them.