53 Summer '06 Practice Finals.

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The real final will have the same structure as these practice finals:

- ∇ Question 1 will have 12 parts (1 on Chapter 10, 1 on Chapters 12 and 13, 3 on Chapter 14, 3 on Chapter 15 and 4 on Chapter 16) which are similar to the Concept Check parts of the book: you should answer *at most six* parts of this questions, for a maximum of 30 points.

  Questions 2, 3, 4 and 5 will be like the unstarred problems on the homework and the 2nd questions on the quizzes. One will be on Chapter 14, one on 15 and two on 16. They wil be worth 8, 8, 9 and 9 points respectively.

- ∇ Questions 6, 7 and 8 will be like the starred problems on the homework and the 3rd questions on the quizzes. One will be on Chapter 14, one on 15 and one on 16. They will be worth 12 points each.

Of course, the actual questions on the final will be different than these, but the level of difficulty should be similar.

Practice Final 1.

Except in question 1, all answers must be justified to receive full credit. Please read the questions carefully before and after answering them and check your work through before turning it in. You have two hours to complete the exam. You are allowed a single side of a sheet of notes, written in your own hand, and writing implements. No other equipment is allowed while doing this exam. Please do not write in red. Good luck!

1. Please answer **at most six** parts of this question. Answer in full sentences and make sure your notation is clear, but no justification is required. [5 points per part.]

   (a) How do you find the slope of a tangent to a parametric curve at a point?

   (b) How do you find the equation of a plane given three (non co-linear) points on it?

   (c) State Clairaut's Theorem.

   (d) Define the linearization of f at (a, b). What is the geometric interpretation of linear approximation?

   (e) Explain how to use the Method of Lagrange multipliers in finding extreme values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$.

   (f) How do you change an integral from rectangular to polar coordinates? Why would you want to?

   (g) Let f be a joint density function of a pair of continuous random variables X and Y. Write expressions for the probability that X lies between a and b and Y lies between c and d and an expression for the expected value of $X + Y$.

(h) How do you change an integral from rectangular to spherical coordinates? Why would you want to?

(i) State the Fundamental Theorem for Line Integrals.

(j) Write an expression for the area enclosed by a curve C in terms of line integrals around C.

(k) If $\mathbf{F}$ is a vector field on $\mathbb{R}^3$, define $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$ (without referring to vector notions). If $\mathbf{F}$ is the velocity field of a fluid, what are the physical interpretations of curl and div?

(l) State the Stokes's Theorem.

2. Find the local maximum, local minimum and saddle points of the function $f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10$.

3. Describe the solid whose volume is given by the integral $\int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$ and evaluate the integral.

4. Let $\mathbf{F} = \langle x, y^2, z^3 \rangle$.

   (a) Is $\mathbf{F}$ the gradient of some function? If so, find it; if not, explain why.

   (b) Is $\mathbf{F}$ the curl of some vector field? If so, find it; if not, explain why.

5. Use Stokes's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle e^{-x}, e^x, e^z \rangle$ and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first octant.

6. Find values of r, s, t which maximize and minimize the distance of the plane $(\mathbf{r} - \langle 1, -1, 2 \rangle) \cdot \langle r, s, t \rangle = 0$ from the origin.

7. Use an appropriate change of variables to evaluate $\iint_D xy \, dA$ where D is the square with vertices $(0, 0), (2, 1), (-1, 2)$.

8. This question is about the vector field $\mathbf{F} = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$.

   (a) Find the line integral of $\mathbf{F}$ around the curve parametrised by $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$, $t \in [0, 2\pi]$.

   (b) Find the scalar curl of $\mathbf{F}$. Cancel it down as far as you can.

   (c) Do your answers to parts (a) and (b) contradict Green's Theorem? If no, explain both why not and why someone might (wrongly) think they do.

Practice Final 2.

Except in question 1, all answers must be justified to receive full credit. Please read the questions carefully before and after answering them and check your work through before turning it in. You have two hours to complete the exam. You are allowed a single side of a sheet of notes, written in your own hand, and writing implements. No other equipment is allowed while doing this exam. Please do not write in red. Good luck!

1. Please answer **at most six** parts of this question. Answer in full sentences and make sure your notation is clear, but no justification is required. [5 points per part.]

   (a) How do you find the area of a region bounded by a polar curve?

   (b) How could you use dot products to find the angle between two vectors?

   (c) If $x = f(x, y)$, what are the differentials dx, dy and dz.

(d) State the Chain Rule for the case where $z = f(x, y)$ and x and y are functions of one variable.

(e) State the second derivatives test.

(f) Suppose a solid object occupies the region E and has density function $\rho(x, y, z)$. Write expressions for the mass and the x-coordinate of the center of mass.

(g) Write an expression for the area of a surface with equation $z = f(x, y)$, $(x, y) \in D$.

(h) If a transformation is given by $x = g(u, v)$, $y = h(u, v)$, what is the Jacobian of the transformation? How is it used in changing variables in an integral?

(i) What is a vector field? Give three examples that have physical meaning.

(j) State Green's Theorem.

(k) What is an oriented surface? Give an example of a non-orientable surface.

(l) State the Divergence Theorem.

2. Find equations of the tangent plane and the normal line to the surface $sin(xyz) = x + 2y + 3z$ at the point $(2, -1, 0)$.

3. Find $\iiint_E yz\, dV$ where E is tetrahedron with vertices $(0, 0, 0), (3, 0, 0), (0, 1, 0), (0, 0, 1)$.

4. Show that $\mathbf{F} = \langle e^y, xe^y + e^z, ye^z \rangle$ is conservative and use this to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the line segment connecting $(2, 0, 3)$ to $(4, 0, 3)$.

5. Use the divergence theorem to calculate $\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle x^2, sin(xyz), e^{z^2 - xy} \rangle$ and S is the surface of the box with vertices $(\pm 1, \pm 2, \pm 3)$.

6. A unit square is cut into four rectangles by two lines parallel to the sides. Find the maximum and minimum values of the sum of the squares of the areas of the smaller rectangles.

7. If X, Y and Z are uniform random variables on $[0, 1]$ find the expectation of the maximum of X, Y and Z. [Hint: Find the "expectation" of X using just the part of the domain on which X is greater than Y and Z. Explain why tripling this gives the right answer.]

8. Find the positively oriented simple smooth curve, C, which maximizes the integral $\int_C (y^3 - y)\, dx - 2x^3\, dy$. [Hint: Use Green's Theorem.]