Math 1a Practice Final, Fall 2004

Note: Each of the questions will be graded out of 10. Show work and give reasons.

1. Find the local maxima and minima and the intervals of increase and decrease of the function
$$f(x) = \frac{2x-5}{x^2-4}.$$

2. Suppose $x^2 + y^2 = r^2$, $x = A/w$, $dy/dt = -2$, $dr/dt = 2$, and $dw/dt = -1/2$. Find dA/dt when $r = 5$, $A = 4$, $w = 1$, and $y > 0$.

3. Evaluate the limit, if it exists (possibly as an infinite limit):

$$\lim_{x\to 1^+} \frac{|x^2 - 3x + 2|}{x^2 - 1}.$$

4. Find a and b so that the function

$$f(x) = \begin{cases} x^4 + 3x^3 + 5x^2 - 2 & \text{if } x \leq -1 \\ ax + b & \text{if } x > -1 \end{cases}$$

is differentiable at $x = -1$.

5. Differentiate the function $f(x) = x^{x^3} \sin x$.

6. If $(x + y)^2 + (y - 4)^2 = 25$, express dy/dx in terms of x and y.

7. What happens when you apply Newton's method to the equation $x^2 + 2x + 4 = 0$ with initial guess $x_0 = 0$? Why would we expect the method to not find a solution of the equation? [Hint: What are the solutions?]

8. **By considering areas**, evaluate $\int_0^a \sqrt{1 - x^2}\, dx$ when $0 < a < 1$ (in terms of a).

9. Find $\frac{d}{dx}\int_1^{x^2+1} \cos(\frac{1}{t})\,dt$.

10. Use a substitution to find $\int \frac{x^2}{(x+1)^{10}}\,dx$.

11. By recognizing it as a Riemann sum, evaluate

$$\lim_{n\to\infty} \frac{\pi}{n}(\sin(\pi/n) + \sin(2\pi/n) + \sin(3\pi/n) + \cdots + \sin(\pi))$$

12. State the Mean Value Theorem for integrals. For $\int_0^1 (x^2 + A)\,dx$, where A is some constant, find a c which satisfies the conclusion.