

You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Inverse Functions

1. The graph of $f(x)$ is given. Draw the graph of $f^{-1}(x)$.
2. Suppose f is one-to-one. How are the domain and range of f related to the domain and range of f^{-1} ?
3. Which of the following functions are one-to-one?
 - (a) x^2
 - (b) x^3
 - (c) $x^3 + 4$
 - (d) $\sin(x)$
 - (e) e^x
 - (f) $e^{(x^2)}$
 - (g) $|x|$
 - (h) $x|x|$
4. Compute the inverses of the functions that are one-to-one in the previous question. Why can't you define the inverse of a function that is not one-to-one?

Trig Functions

1. What are all of the trig identities that you can think of? Which are important for the class?
2. For each of the following trigonometric functions, graph the function, highlight its usual principal domain, and draw the inverse function corresponding to that choice of the principal domain.
 - (a) $\sin x$
 - (b) $\cos x$
 - (c) $\tan x$
3. Simplify the following expressions, using either trig identities or by drawing a triangle.
 - (a) $\sin(\tan^{-1} x)$.
 - (b) $\cos(\sin^{-1} x)$.
 - (c) $\cot(\tan^{-1} x)$.
 - (d) $\tan(\sin^{-1} x)$.

Limits

1. Find each of the following limits (or explain why they do not exist), where f is the function whose graph is shown below:

(a) $\lim_{x \rightarrow 0} f(x)$

(b) $\lim_{x \rightarrow 1} f(x)$

(c) $\lim_{x \rightarrow -1} f(x)$

(d) $\lim_{x \rightarrow 3^+} f(x)$

(e) $\lim_{x \rightarrow 3^-} f(x)$

(f) $\lim_{x \rightarrow 3} f(x)$

2. Sketch a graph of a function with *all* of the following properties:

• $\lim_{x \rightarrow 0^+} f(x) = 2$

• $\lim_{x \rightarrow 3} f(x) = 0$

• $f(0) = 0$

• $\lim_{x \rightarrow 0^-} f(x) = -1$

• $\lim_{x \rightarrow -1} f(x) = 4$

• $f(3) = 1$

3. Let L and a be real numbers. Sketch a graph of a function where:

(a) $\lim_{x \rightarrow a} f(x) = L$ and $f(a) = L$

(b) $\lim_{x \rightarrow a} f(x) = L$ but $f(a) \neq L$

(c) $\lim_{x \rightarrow a^+} f(x)$ and $\lim_{x \rightarrow a^-} f(x)$ both exist, but $\lim_{x \rightarrow a} f(x)$ does not

Inequalities and Intervals

1. Suppose δ is some positive real number and a is some real number. Express each of the following sets of numbers

• Using inequalities

• Using absolute values

• Using intervals

(a) All the numbers that are within δ of 0

(b) All the numbers that are within δ of a

(c) All the numbers that are within δ of a but not equal to a