Qualifying Exam Syllabus

Bradley M. Froehle
(bfroehle@math.berkeley.edu)

December 18, 2009, 1–4pm, 961 Evans Hall

Committee:

Craig Evans (Chair)
Jon Wilkening
John Strain
Larry Karp (Agriculture and Resource Economics)

Partial Differential Equations (Classical Analysis)

Laplace's Equation: fundamental solution; mean value formulas; properties of harmonic functions; Dirichlet's principle.
Heat Equation: fundamental solution; initial-value and non-homogeneous problems; mean value formula; regularity; energy methods.
Wave Equation: solution by spherical means; energy methods; non-homogeneous problem.
Sobolev Spaces: definition; weak derivatives; approximation by smooth functions; extensions; traces; Gagliardo-Nirenberg-Sobolev inequality; Poincaré's inequality.
Second-Order Elliptic Equations: definition; weak solutions; existence theorems; Lax-Milgram theorem; Fredholm alternative; regularity; maximum principles; eigenvalue and eigenfunctions.
Calculus of Variations: first variation; Euler-Lagrange equation; second variation; existence of minimizers; regularity; null Lagrangians; constraints; critical points;
Hamilton-Jacobi Equations: definition; viscosity solution; uniqueness; control theory; dynamic programming; Hamilton-Jacobi-Bellman equation; Hopf-Lax formula.

References:
Partial Differential Equations. Lawrence C. Evans
Applied Computational Economics and Finance. Mario J. Miranda & Paul L. Fackler

Numerical ODE and PDE (Applied Mathematics)

Ordinary differential equations:

Basic notions: convergence; consistency; A-, A(α)-, B-, and L-stability; region of absolute stability; local truncation error; implicit and explicit schemes.
Runge-Kutta methods: Order conditions; stability; collocation and implicit methods; Gaussian quadrature.
Multistep methods: Order conditions; stability; First and second Dahlquist barriers; Adams-Bashforth and Adams-Moulton; backward differentiation formula.
Error estimation & stepsize control: embedded Runge-Kutta; paired Adams-Bashforth/Adams-Moulton

methods.

Partial differential equations:

Finite differences (for the heat and wave equations): stability; consistency; convergence; local truncation error; CFL condition; von Neumann stability analysis; basic schemes (leapfrog, Crank-Nicholson, Lax-Friedrichs, Lax-Wendroff, upwinding).
Finite elements: definition; Galerkin methods; weak formulation; existence of solutions; error analysis.
Meshes: definition; Deluanay triangulation and refinement; advancing front; structured mesh generation.

References:
A First Course in the Numerical Analysis of Differential Equations, 2nd ed. Arieh Iserles
Theory and Practice of Finite Elements. Alexandre Ern & Jean-Luc Guermond

Minor Topic: Numerical Linear Algebra (Applied Mathematics)

Basic notions: condition number; backward stability; floating point arithmetic.
Basic algorithms & analysis: Horner's rule (polynomial evaluation); matrix multiplication; counting floating point operations and memory transfer; parallel computing; divide-and-conquer algorithms.
Gaussian elimination: LU; Cholesky; pivoting; error analysis; solving linear systems.
Linear Least Squares: normal equations; QR; singular value decomposition; Givens rotations; Householder reflections; rank deficient least squares.
Eigenvalues: Rayleigh quotient; Schur form; QR Iteration; Hessenberg reduction; tridiagonal reduction; bidiagonal reduction.

References:
Applied Numerical Linear Algebra. James W. Demmel