BMS Short Course **Tropical Geometry**
Exercises for Thursday and Friday, September 13-14

(1) Let I be the ideal generated by three general homogenous linear forms in $\mathbf{C}[x_1, x_2, x_3, x_4, x_5]$. Describe the tropical variety $\mathcal{T}(I)$ and find a minimal tropical basis for I. What changes if the field $\mathbf{C}$ of complex numbers gets replaced by the field $K = \mathbf{C}\{\{t\}\}$ of Puiseux series?

(2) Draw a general tropical plane in tropical 4-space projective $\mathbf{TP}^4$. How many vertices, edges and polygons does your plane have? How many combinatorial types of tropical planes in $\mathbf{TP}^4$ are there altogether?

(3) How many edges does the tropical Grassmannian $\mathcal{G}_{2,7}$ have?

(4) What is the dimension of the prevariety $pre\mathcal{G}_{3,7}$?

(5) Let M be the graphic matroid associated with the complete bipartite graph $K_{2,3}$. Its bases of M are collections of edges in $K_{2,3}$ that form spanning trees. Draw the Bergman complex $\mathcal{T}(M)$. What about $K_{3,3}$?

(6) Desribe a necessary and sufficient condition for a tropical linear system of equations $A \odot x = b$ to be solvable.

(7) Find the image of the tropical linear map $\mathbf{R}^3 \to \mathbf{R}^3$ given by the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

(8) How many combinatorial types of tropical triangles in $\mathbf{TP}^2$ are there?

(9) Let X be a curve in $\mathbf{C}^3$ given by a parametrization $t \mapsto (f(t), g(t), h(t))$ where f, g, h are polynomials in one variable. The tropical curve $\mathcal{T}(X)$ is a one-dimensional weighted fan in $\mathbf{R}^3$. Describe it in terms of f, g, h.

(10) Compute the discriminant of a general quintic in one variable,

$$f(t) \quad = \quad x_5 \cdot t^5 \,+\, x_4 \cdot t^4 \,+\, x_3 \cdot t^3 \,+\, x_2 \cdot t^2 \,+\, x_1 \cdot t \,+\, x_0.$$

Compute and draw the Newton polytope of this discriminant.

(11) Find the equation of a tropical cubic curve through the eight points

$$(0,14),\ (5,12),\ (10,10),\ (15,8),\ (20,6),\ (25,4),\ (30,2),\ (35,0).$$

Propose a description of the set of all cubic curves through these points.