

Math 16A (Bernd Sturmfels), **Final Exam**

Friday, December 16, 12:30 p.m.–3:30 p.m.

This exam is closed book. You must show all your all work to get credit. Each problem is worth 10 points, for a total of 100 points. As calculators are not permitted, you do not need to evaluate expressions like e^4 or $\ln(1/2)$.

- (1) Suppose that the weight of a cancerous tumor after t weeks is $W(t) = 0.1 \cdot t^2$ grams. What is the average rate of growth of the tumor during the fifth week, that is, from time $t = 4$ to $t = 5$? What is the (instantaneous) rate of growth of the tumor at time $t = 4$?
- (2) Determine all functions $f(x)$ whose second derivative equals $1/x^2$. To keep things simpler, you may assume that $x > 0$.
- (3) Find the tangent line to the graph of $xy + y^3 = 14$ at the point $(3, 2)$.
- (4) Compute the derivatives of the following two functions:
 $f(x) = \ln(\ln(x))$ and $g(x) = (x+1)^2(x+2)^3(x+3)^5(x+4)^7(x+5)^{11}$.
- (5) One hundred gram of radioactive radium having a half-life of 1600 years is placed in a concrete vault. What will be the average amount of radium in the vault over the next 1000 years?
- (6) Find a demand function whose elasticity is constant and equal to 2.
- (7) Compute the following two definite integrals

$$\int_{-1}^2 \frac{3}{(x+2)^4} dx \quad \text{and} \quad \int_0^1 \frac{7}{e^{4x}} dx.$$

- (8) Describe an application of the differential equation $y' = ky(M - y)$. In your application, what do the constants k and M mean? Write down a function $y = f(t)$ which satisfies this differential equation.
- (9) Determine positive values of x , y , and z that maximize the product $Q = xyz$ if $x + y = 1$ and $y + z = 2$. What is the maximum value?
- (10) The sequence of *Fibonacci numbers* $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots$ is defined as follows. If $f(n)$ is the n^{th} Fibonacci number then
 $f(0) = 0$, $f(1) = 1$ and $f(n) = f(n-1) + f(n-2)$ for all $n \geq 2$.
Write the Fibonacci numbers as a sum of two exponential functions, i.e., find four constants C_1, C_2, a and b such that

$$f(n) = C_1 \cdot a^n + C_2 \cdot b^n \quad \text{for all } n \geq 0.$$