

QUALIFYING EXAM SYLLABUS

for Dario Beraldo

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Complex geometry.

- Complex manifolds: definition and examples.
- Line bundles and divisors.
- Kähler manifolds, Kähler identities.
- Hodge decomposition (assuming elliptic regularity).
- Hard Lefschetz theorem. Hodge index theorem.
- Lefschetz theorem on $(1, 1)$ -classes and Hodge conjecture.
- Hermitian vector bundles and Serre duality.
- Connections, curvature, Chern classes.
- Hirzebruch-Riemann-Roch theorem.
- Lefschetz theorem on hyperplane sections.
- Kodaira vanishing and embedding theorems.

Representation theory.

- Lie algebras: definition and examples.
- Nilpotent and solvable Lie algebras. Engel's and Lie's theorems.
- Killing form and Cartan's criterion.
- Simple and semisimple Lie algebras.
- Roots, Weyl group, Cartan matrices, Dynkin diagrams.
- Universal enveloping algebra. Poincaré-Birkhoff-Witt theorem.
- Theorem of the Highest weight. Verma modules.
- Dixmier's lemma. Complete reducibility.
- Casimir elements. Harish-Chandra isomorphism.
- Weyl character formula, Weyl dimension formula.
- Kostant multiplicity formula.

\mathcal{D} -modules.

- The sheaf \mathcal{D} on a smooth algebraic variety. Principal symbols.
- Coherent \mathcal{D} -modules. Flat connections.
- Basic functorial operations with \mathcal{D} -modules.
- Spencer resolution.
- Good filtrations and characteristic variety of a coherent \mathcal{D} -module.
- Holonomic \mathcal{D} -modules.
- Solution complexes and de Rham functors. Constructible sheaves.
- Regular holonomic \mathcal{D} -modules and Riemann-Hilbert correspondence.

References.

D. Huybrechts, Complex geometry

A. Knapp, Lie groups beyond an introduction

R. Hotta, K. Takeuchi, T. Tanisaki, \mathcal{D} -modules, perverse sheaves and representation theory