Worksheet For DIS 201 /210 Chapter 3 Determinants

Problem 1: Calculate the determinants

(1) $\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{vmatrix}$ (2) $\begin{vmatrix} 2 & 5 & 4 & 1 \\ 4 & 7 & 6 & 2 \\ 6 & -2 & -4 & 0 \\ -6 & 7 & 7 & 0 \end{vmatrix}$ (3) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

(Vandemonde matrices)

Problem 2. Use determinants to decide if the set of vectors are linearly independent

$\begin{bmatrix} 8 \\ -4 \\ -6 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}$

Problem 3: Use Cramer's rule to solve the equation $\begin{cases} 4x_1 + x_2 = 6 \\ 5x_1 + 2x_2 = 7 \end{cases}$

Problem 4: if $\underline{u} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ $\underline{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ compute the area of the parallelogram determined by $\underline{u}$, $\underline{v}$, $u+v$, 0.

now $T(x) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \underline{x}$ calculate $T\underline{u}$, $T\underline{v}$ $T(\underline{u}+\underline{v})$. $T(0)$

what is the area of the new parallelogram?

Is this true $\text{Area}(T(s)) = |\det T| \, \text{Area}(s)$ where s is the parallelogram determined by $\underline{u}$. $\underline{v}$. $\underline{u+v}$. 0

Problem 4: Solve the equation

$\det(A - \lambda I) = 0$ $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ λ is the ~~at~~ unknown.

Problem 5: Compute $\text{Adj}(A)$ $A = \begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Problem 6: Quick Questions

(1) $\det A = 3$ then what is $\det kA$? A $n \times n$, what is $\det(A^{-1})$?

and what is $\det(\text{adj } A)$?

(2)

Compute $\det B^5$, $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

(3) True or False: Give explanation or examples. or Counter examples.

(a) $\det(-A) = \det A$?

(b) $\det(A^T) = \det A$?

(c) $\det(A+B) = \det A + \det B$

(d) $\det(A^T A) \geq 0$

(e) If $U^T U = I$ then $\det U = \pm 1$?

(f) $\det A^4 = 0$, then A not invertible ?

(g) P invertible. then $\det(PAP^{-1}) = \det A$?

(h) Columns of A linearly dependent, then $\det A = 0$.

(4) what is the inverse of ~~det~~ adj A if $\det A \neq 0$.

and what is the $\det(\text{adj } A)$ interms of $\det(A)$?

Problem 7:

we know the area of a circle $x_1^2 + x_2^2 = 1$ is π.

If you take the transformation $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

then what is the equation satisfied by y_1, y_2 ?

And what is the area of the image of the circle?

(Also see P210. problem 31 for 3-dimension case)