

Name: _____

SID: _____

Problem 1: (4 points)

Given $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$

Find basis for $\text{col}(A)$ and basis for

$\text{nul}(A)$ (each for 2 points)

Problem 2: (3 points) calculate the determinant

$$\begin{vmatrix} 0 & 4 & 0 & 8 \\ 5 & 0 & 1 & 4 \\ 2 & 0 & 5 & 7 \\ 0 & 4 & 2 & 0 \end{vmatrix}$$

Problem 3 (Proofs) (4 points)

(i) Show that: if we know $A = PAP^T$ A, P are $n \times n$ matrix
then either $\det A = 0$ or $\det P = \pm 1$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) There is a theorem says $\text{rank } A = \text{rank } (A^T)$, use this, together with rank theorem you have learned to show that

For a matrix $A_{m \times n}$ $\dim(\text{nul } A) - \dim(\text{nul } A^T) = n - m$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rank(A) = rank(A^T) = rank(A)