Solution To Quiz 8 DIS 210

Problem 1 : (1) $Tp(t) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(2) $T1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$Tt = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ So matrix is $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$Tt^2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Problem 2 : (1) $\begin{vmatrix} -3 & 1 & -1 \\ 7 & 1 & 3 \\ 5 & 1 & -2 \end{vmatrix} = \begin{vmatrix} -3 & 1 & -1 \\ 10 & 0 & 4 \\ 8 & 0 & -1 \end{vmatrix} = (-1) \begin{vmatrix} 10 & 4 \\ 8 & -1 \end{vmatrix}$

$= (-1)(-10-32) \neq 0$

So $\begin{pmatrix} -3 \\ 7 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ are linearly independent

Hence $\vec{y}$ is not in $W = \text{span}\{u, v\}$

(2) $\text{Proj}_W y = \dfrac{y \cdot u}{u \cdot u} u + \dfrac{y \cdot v}{v \cdot v} v$

$= \dfrac{-3+7+5}{3} u + \dfrac{3+21-10}{1+9+4} v = 3u + v$

this is the best approximation

(3) $y - \text{Proj}_W y = y - 3u - v = \begin{pmatrix} -3 \\ 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ -4 \end{pmatrix}$

so distance of y and W is $\|y - \text{Proj}_W y\| = \sqrt{5^2+1^2+4^2} = \sqrt{42}$

(4) $\cos\theta = \dfrac{y \cdot u}{u \cdot u} = \dfrac{-3+7+5}{\sqrt{3^2+7^2+5^2}\sqrt{3}} = \dfrac{9}{\sqrt{249}}$ $\theta = \cos^{-1}\dfrac{9}{\sqrt{249}}$

Problem 3 : write $||u-v||^2 = ||u+v||^2$ in the form of inner product

$(u-v)\cdot(u-v) = (u+v)\cdot(u+v)$

$u\cdot u - v\cdot u - u\cdot v + v\cdot v = u\cdot u + v\cdot u + u\cdot v + v\cdot v$

Notice $u\cdot v = v\cdot u$

hence we get $-2u\cdot v = 2u\cdot v$ so $4u\cdot v = 0$

hence $u\cdot v = 0$ $u \perp v$

Problem 4 (1) if $Ax = \lambda x$ then $A^2x = A(Ax) = A(\lambda x) = \lambda Ax = \lambda^2 x$

So $(5I + A^2)x = 5x + A^2x = 5x + \lambda^2 x$

$= (5+\lambda^2)x$

x is the eigenvector. corresponds to the eigenvalue $5+\lambda^2$

(2) Consider the example $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

It has only eigenvalue 1. but 1 has only one corresponding eigenvector

Hence not diagonalizable