Solution To Quiz 6 D/S 2/0

Problem 1

(1) $X = B[x]_B$ Solve the equation $\begin{bmatrix} 3 & -4 & -5 \\ -5 & 6 & 7 \end{bmatrix}$

$$\to \begin{bmatrix} 3 & -4 & -5 \\ 0 & -2 & -4 \end{bmatrix} R1\times 5 + R2 \times 3$$

$$\to \begin{bmatrix} 3 & -4 & -5 \\ 0 & 1 & 2 \end{bmatrix}$$

So $[x]_B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\to \begin{bmatrix} 3 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

(2) $X = B[x]_B = \begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} -5 \\ 7 \end{bmatrix}$

$$= \begin{bmatrix} -15 & -28 \\ 25 & +42 \end{bmatrix} = \begin{bmatrix} -43 \\ 67 \end{bmatrix}$$

Problem 2:

$$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 2 & -1 & 1 & -6 & 8 \\ -7 & 8 & 10 & -3 & 10 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 3 & 9 & -12 & 12 \\ 0 & -6 & -8 & 24 & -24 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 1 & 3 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So basis are $(1, -2, -4, 3, -2)$ $(0, 1, 3, -4, 4)$

and $\dim \text{nul } A + \text{rank } A = 5$

$\text{rank } A = 2$

so $\dim \text{nul } A = 3$

Problem 3

(1) True if they span the space that means they are the basis

so $\dim V = p$

if they do not span the space. we need more vectors to form the basis so $\dim V > p$

Hence $\dim V \geq p$

(2) True you look at the three row operations

change 2 rows $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \sim \begin{pmatrix} a_2 \\ a_1 \\ a_3 \end{pmatrix}$

multiply a constant $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \sim \begin{pmatrix} \lambda a_1 \\ a_2 \\ a_3 \end{pmatrix}$

add a multiple of another row $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \sim \begin{pmatrix} a_1 + \lambda a_2 \\ a_2 \\ a_3 \end{pmatrix}$

and you find that the row space are the same

(2) False counter example $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

A B

$\text{col}(A) = \text{Span}\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$ $\text{col}(B) = \{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\}$ not the same!